UNIT – IV
AC CIRCUIT ANALYSIS FOR NON-SINUSOIDAL EXCITATION

Topics: Fourier theorem- Trigonometric and exponential form of Fourier series – conditions of symmetry- line spectra and phase angle spectra- Analysis of Electrical Circuits to Non sinusoidal periodic waveforms.

INTRODUCTION TO FOURIER THEOREM:

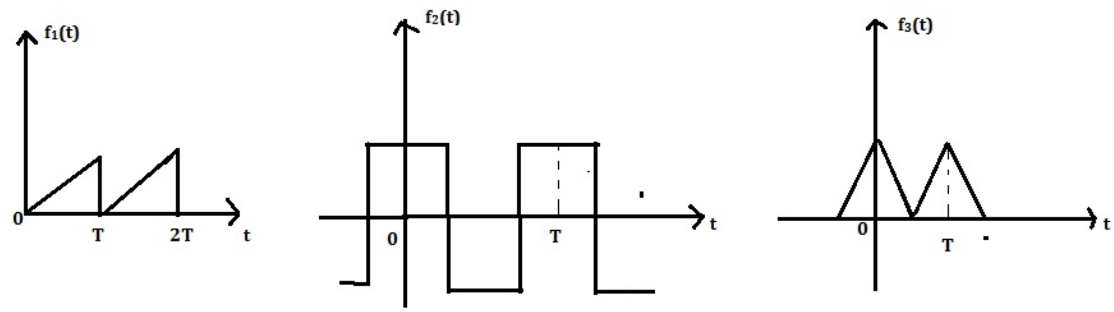
Fourier series gives the representation of any periodic non-sinusoidal waveforms in terms of sine cosine and their harmonics. The ideas and techniques of Fourier series can be extended to non-periodic signals. The signal representation for such non-periodic signals is given by Fourier integrals and Fourier transforms.

Fourier analysis deals with Fourier series and Fourier transforms and has several applications in Mathematics, Science and Engineering particularly in the area of communications and signal processing. Any periodic signal can be represented as the sum of a finite or infinite number of sinusoidal functions, the responses of linear systems to non-sinusoidal excitations can he determined by applying the superposition integral.

PERIODIC SIGNAL :

A signal f (t) is said to be periodic of period T, if f(t) = f (t+T), for all T.

EX:-



The fundamental frequency of the periodic signal is given by $\omega_o = 2\Pi/T$. If 'n' is any integer, then $n\omega_o$ is called n^{th} harmonic of the fundamental frequency ω_o.

DIRICHLET CONDITIONS

The Fourier series exists for the given periodic function if it satisfies the following properties.

1) The function f(t) is single valued everywhere.

2) The Integral $\int_{t0}^{t0+T} |f(t)|\, dt$ exists for any t_0.

3) f(t) has a finite number of discontinuities in any one time period.

4) f(t) has finite number of maxima and minima in any one period.

Note:

$$1)\int_0^T \sin m\omega_o t dt = 0, \forall m \neq 0$$

$$2)\int_0^T \cos n\omega_o t dt = 0, \forall n \neq 0$$

$$3)\int_0^T \sin m\omega_o t \cos n\omega_o t dt = 0, \forall m, n$$

$$4)\int_0^T \sin m\omega_o t \sin n\omega_o t dt = 0, \forall m \neq n$$

$$5)\int_0^T \cos m\omega_o t \cos n\omega_o t dt = 0, \forall m \neq n$$

$6) if$

$m = n$

$$i)\int_0^T \sin^2 m\omega_o t dt = \frac{T}{2} \forall m$$

$$ii)\int_0^T \cos^2 m\omega_o t dt = \frac{T}{2} \forall n$$

REPRESENTATION OF FOURIER SERIES :

The Fourier series can be represented in two ways.

They are (i) Trigonometric Fourier Series

(ii) Exponential Fourier series

TRIGONOMETRIC FOURIER SERIES

For any periodic function f(t), the Fourier series can be represented as

$$f(t) = a_0 + a_1 \cos \omega_0 t + a_2 \cos \omega_0 t + ... + b_1 \sin \omega_0 t + b_2 \sin \omega_0 t + ...$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \, cos n\omega_0 t + b_n \sin n\omega_0 t)$$

The constants a_0, a_n, b_n are called fourier coefficients.

The process of determining a_0, a_n, b_n is called **Fourier series analysis.**

EVALUATION OR DERIVATION OF FOURIER COEFFICIENTS

The Fourier series can be represented by

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \, cos n\omega_0 t + b_n \sin n\omega_0 t) \quad \text{—— (i)}$$

Integrating eq(i) with limits 0 to T

$$\int_0^T f(t)dt = \int_0^T a_0 dt + \sum_{n=1}^{\infty}\int_0^T a_n \cos n\omega_0 t dt + \sum_{n=1}^{\infty}\int_0^T b_n \sin n\omega_0 t dt$$

$$\int_0^T f(t)dt = a_0 T + 0 + 0$$

$$\therefore a_0 = \frac{1}{T}\int_0^T f(t)dt$$

Multiply eq(i) by $\cos k\omega_0 t$ and integrating with the limits 0 to T

$$\int_0^T f(t)\cos k\omega_0 t dt = \int_0^T a_0 \cos k\omega_0 t dt + \sum_{n=1}^{\infty}\left\{\int_0^T a_n \cos n\omega_0 t \cos k\omega_0 t dt + \int_0^T b_n \sin n\omega_0 t \cos k\omega_0 t dt\right\}$$

If k=n

$$\int_0^T f(t)\, cos n\omega_0 t\, dt = 0 + a_n \frac{T}{2} + 0$$

$$\therefore a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_0 t dt$$

Multiply eqn (i) by $\sin k\omega_0 t$ and integrating with the limits 0 to T

$$\int_0^T f(t)\sin k\omega_0 t dt = \int_0^T a_0 \sin k\omega_0 t dt + \sum_{n=1}^{\infty}\left\{\int_0^T a_n \cos n\omega_0 t \sin k\omega_0 t dt + \int_0^T b_n \sin n\omega_0 t \sin k\omega_0 t dt\right\}$$

If k=n

$$\int_0^T f(t)\, sin\, n\omega_0 t\, dt = 0 + 0 + b_n \frac{T}{2}$$

$$\therefore b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_0 t dt$$

EXPONENTIAL FOURIER SERIES
(Or)
COMPLEX FORM OF FOURIER SERIES

We know, the trigonometric Fourier series is

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t) \quad \text{——— (i)}$$

[$e^{jx} = \cos x + j \sin x$ & $e^{-jx} = \cos x - j \sin x$

$$\cos x = \frac{e^{jx} + e^{-jx}}{2} \quad \& \quad \sin x = \frac{e^{jx} - e^{-jx}}{2j} \;]$$

From the above analysis, eq(i) becomes

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left\{ a_n \left[\frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2} \right] + b_n \left[\frac{e^{jn\omega_0 t} - e^{-jn\omega_0 t}}{2j} \right] \right\}$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left\{ \left[\frac{a_n - j\, b_n}{2} \right] e^{jn\omega_0 t} + \left[\frac{a_n + j\, b_n}{2} \right] e^{-jn\omega_0 t} \right\} \quad \text{—— (ii)}$$

Consider $F_0 = a_0$ $\quad F_n = \frac{1}{2}(a_n - j\, b_n)$ $\quad F_{-n} = \frac{1}{2}(a_n + j\, b_n)$

Substitute these values in eq(ii)

$$f(t) = F_0 + \sum_{n=1}^{\infty} \{ F_n\, e^{jn\omega_0 t} + F_{-n}\, e^{-jn\omega_0 t} \}$$

$$f(t) = \sum_{n=0}^{\infty} F_n\, e^{jn\omega_0 t} + \sum_{n=1}^{\infty} F_{-n}\, e^{-jn\omega_0 t}$$

$$f(t) = \sum_{n=0}^{\infty} F_n\, e^{jn\omega_0 t} + \sum_{n=-1}^{-\infty} F_n\, e^{jn\omega_0 t}$$

$$f(t) = \sum_{n=-\infty}^{-\infty} F_n\, e^{jn\omega_0 t} \quad \text{——— (iii)}$$

The eq(iii) is called as exponential Fourier series. The exponential Fourier series coefficient $\boldsymbol{F_n}$ is given by

$$\text{Fn} = \frac{1}{2}(a_n - j\, b_n)$$

$$\Rightarrow\ F_n = \frac{1}{2}\left[\frac{2}{T}\int_0^T f(t)\, cosn\omega_0 t\ \ dt - j\ \frac{2}{T}\int_0^T f(t)\, sinn\omega_0 t\ \ dt\right]$$

$$F_n = \frac{1}{T}\int_0^T f(t)(\cos n\omega_0 t - j\ sinn\omega_0 t)dt$$

$$\boldsymbol{F_n = \frac{1}{T}\int_0^T f(t) e^{-jn\omega_0 t} dt}$$

The coefficient $\boldsymbol{F_n}$ is called line spectrum or discrete spectrum. It has magnitude line spectrum ($|F_n|\ Vs\ n$ and even function n) and phase spectrum (Θ = ∠ $\boldsymbol{F_n}$ $Vs\ n$ and odd function n).

CONDITIONS OF SYMMETRY

The functions considered so far had period 2π. In several applications, periodic functions will generally have other periods.

If a functions $x(t)$ of period $2T$ has a Fourier series, then this series is

$$x(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi}{T}\omega t + b_n \sin\frac{n\pi}{T}\omega t\right)$$

With the Fourier coefficients of $x(t)$ given by

$$a_0 = \frac{1}{2T}\int_{-T}^{T} x(t)\, d(\omega t)$$

$$a_n = \frac{1}{T}\int_{-T}^{T} x(t)\cos\left(\frac{n\pi\omega t}{T}\right) d(\omega t) \text{ for } n = 1, 2\ldots$$

$$b_n = \frac{1}{T}\int_{-T}^{T} x(t)\sin\left(\frac{n\pi\omega t}{T}\right) d(\omega t) \text{ for } n = 1, 2\ldots$$

WAVEFORM SYMMETRIES

There are 4 types of wave symmetries.

1) Even symmetry (mirror symmetry):

A function f(t) is said to be even

If $f(t) = f(-t)$.

The sum of two or more even functions is an even function. Even functions are always symmetrical about vertical axis.

If a function having even symmetries $f_e(t)$,then

$$\int_0^T f_e(t)dt = \int_{-\frac{T}{2}}^{\frac{T}{2}} f_e(t)dt = 2\int_0^{T/2} f_e(t)dt$$

For even functions

(i) Coefficient a_0 may or may not be zero.

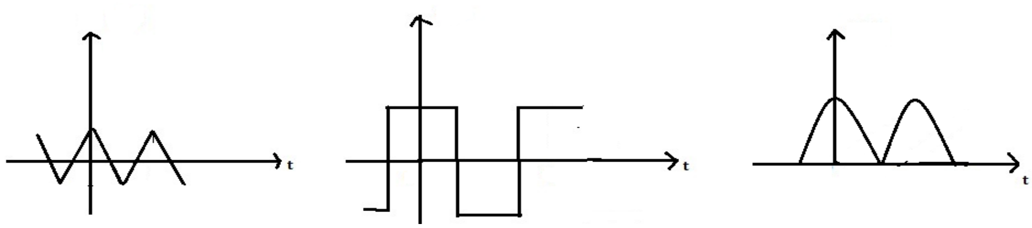
(ii)Coefficient an is given by

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega_0 t\ dt$$

(iii) Coefficient $b_n = 0$

Then the Fourier series expansion of an even function contains only cosine terms.

EX:-



2) Odd symmetry (Rotation Symmetry):

A function f (t) is said to be odd

if $f(t) = -f(-t)$.

The sum of two or more odd functions is an odd function. The product of two odd functions is an even function. The odd functions have diagonal symmetry.

If a function having odd symmetry $f_o(t)$, Then

$$\int_0^T f_o(t)dt = \int_{-\frac{T}{2}}^{\frac{T}{2}} f_o(t)dt = 2\int_0^{T/2} f_o(t)dt$$

For Odd functions

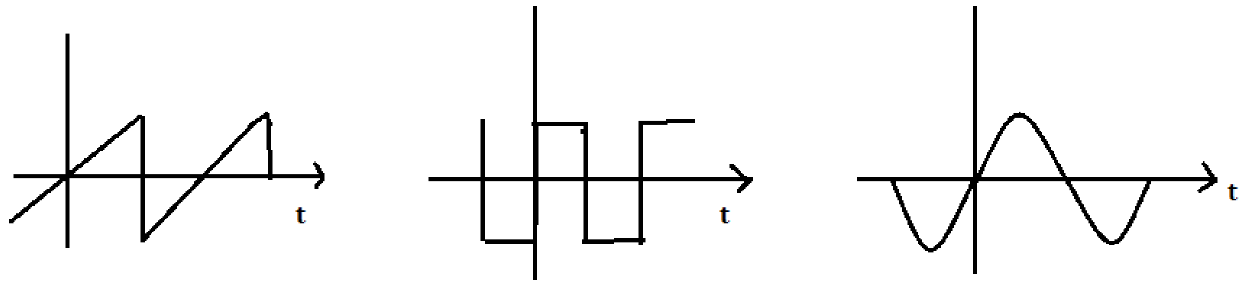
(i) Coefficients a_0, a_n are equal to zero.

(ii) Coefficient bn is given by

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\, sinn\omega_0 t \; dt$$

Then the Fourier series expansion of an odd function contains only sine terms.

EX:-



3) **<u>Half wave symmetry</u>** :

A function f(t) is said to be have half wave symmetry,

If $f(t) = -f(t \pm \frac{T}{2})$

Thus the function is neither completely even nor completely odd or both.

For such functions

(i) Coefficient a_0=0

(ii) Coefficients a_n & b_n are given by

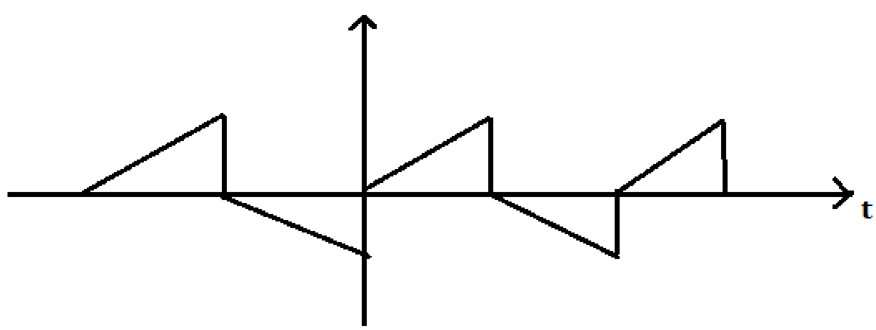
$$a_n = \frac{4}{T}\int_0^{T/2} f(t) cosn\, \omega_0 tdt \quad \text{, if 'n' is odd}$$

$$b_n = \frac{4}{T}\int_0^{T/2} f(t) sinn\, \omega_0 tdt \quad \text{, if 'n' is odd.}$$

(iii) If 'n' is even, a_n=b_n=0.

Thus, the Fourier series expansions of a periodic function with half wave symmetry contain only odd harmonics.

EX:-



4) **<u>Quarter wave symmetry</u>** :

A function f(t) is said to be have quarter wave symmetry,

If f(t) = f(-t) or f(t) = -f(-t)

And f(t) = - f $(t \pm \frac{T}{2})$.

Thus a function having quarter wave symmetry has both, even or odd symmetry as well as half-wave symmetry.

For such functions

Case (i): If f(t) = -f(-t) and f(t) = - f $(t \pm \frac{T}{2})$

(i) Coefficient $a_0=0$

(ii) Coefficient $a_n=0$

(iii) Coefficient b_n is given as

$$b_n = \frac{8}{T}\int_0^{T/4} f(t)\sin n\,\omega_0 t\,dt \text{ , n is odd.}$$

Case(ii): If f(t) = f(-t) and f(t) = - f $(t \pm \frac{T}{2})$

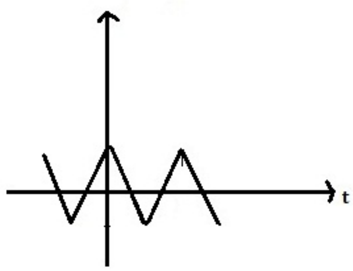
(i) Coefficient $a_0=0$

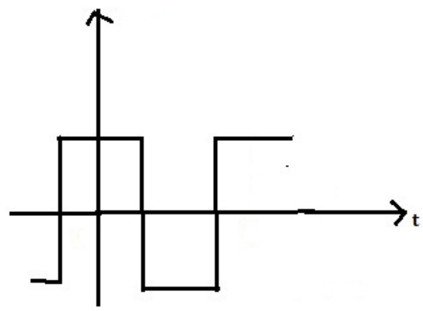
(ii) Coefficient $b_n=0$

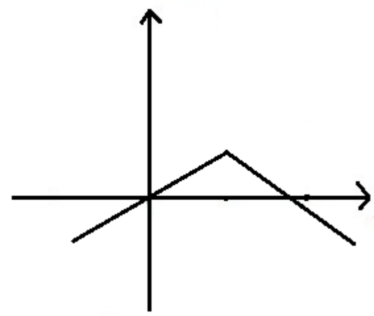
(iii) Coefficient a_n is given as

$$a_n = \frac{8}{T}\int_0^{T/4} f(t)\cos n\,\omega_0 t\,dt \text{ , n is odd.}$$

EX:-







Representation of Any Function in Even and Odd parts

Any general function can be represnted as

$$f(t) = f_e(t) + f_o(t) \quad \text{............ (1)}$$

where $f_e(t)$ is even part of f(t)

$f_o(t)$ is odd part of f(t)

Then $f(-t) = f_e(-t) + f_o(-t)$

$$f(-t) = f_e(t) - f_o(t) \quad \text{............. (2)}$$

by solving eq(1) & eq(2)

$$f_e(t) = \frac{1}{2}[f(t)+f(-t)]$$

and $f_o(t) = \frac{1}{2}[f(t)-f(-t)]$.

LINE SPECTRA AND PHASE SPECTRA
(Or)
AMPLITUDE AND PHASE SPECTRUM

From the trigonometric Fourier series,

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$
$$= A_0 + \sum_{n=1}^{\infty} A_n \cos (n\omega t - \phi_n)$$

where, $A_0 = a_0$, $A_n = \sqrt{a_n^2 + b_n^2}$; $\quad \phi_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)$

Also, for exponential form, C_n is complex and we may write it as,

$$C_n = |C_n| e^{j\phi_n} \text{ and } |C_n| = \frac{1}{2}\sqrt{a_n^2 + b_n^2} = \frac{A_n}{2} \text{ and } \phi_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)$$

The quantities A_n and ϕ_n are called *the amplitude and the phase of the* n^{th} *harmonic*, respectively.

- Variation of A_n with n (or $n\omega$) is known as *the amplitude spectrum or Frequency – spectrum.*
- Variation of ϕ_n with n (or $n\omega$) is known as *the phase- spectrum* of the signal.

As both A_n and ϕ_n occurs at discrete values of the frequency, i.e., n = 1, 2, 3, etc. these spectra are called *Line spectra*.

Since $|C_n| = \frac{A_n}{2}$; there is a scale factor of ½ for the amplitude spectrum for exponential form for the Fourier series compared to the trigonometric form for all lines except the one for $n = 0$. Also, in the case of exponential form spectral lines are drawn for both for positive and negative values of n.

ANALYSIS OF ELECTRICAL CIRCUITS TO NON-SINUSOIDAL PERIODIC WAVEFORMS

AVERAGE VALUE OF A PERIODIC COMPLEX WAVE

Let the Fourier series representation of a periodic complex wave be,

$$v(t) = V_0 + \sum_{n=1}^{n} V_{nm} \sin(n\omega t + \phi_n)$$
$$= V_0 + V_{1m} \sin(\omega t + \phi_1) + V_{2m} \sin(2\omega t + \phi_2) + \ldots\ldots + V_{nm} \sin(n\omega t + \phi_n)$$

The average value of the periodic wave is given by

$$V_{\text{avg}} = \frac{1}{T}\int_0^T v(t)\, dt$$

Since the integration of any sine function between $T = 0$ to $T = 2\pi$ is zero,

$$V_{avg} = V_0 + 0 + 0 + \cdots$$
$$= V_0$$

Hence, the average value of any periodic complex wave is the constant term of the Fourier series representing the periodic wave.

EFFECTIVE VALUE OF A PERIODIC NON-SINUSOIDAL WAVE

A periodic non-sinusoidal waveform of current passing through a resistor results in a power which is determined by the effective or RMS value of the wave.

The effective value of any wave is $\sqrt{\frac{1}{T}\int_0^T f(t)^2 dt}$

Consider the periodic non-sinusoidal function can be represented by

$f(t) = a_0 + a_1 \cos\omega t + a_2 \cos 2\omega t + \ldots\ldots\ldots + b_1 \sin\omega t + b_2 \sin 2\omega t + \ldots\ldots$

Then the effective value becomes

$$\mathbf{F_{eff} = F_{rms} = \sqrt{a_0^2 + \frac{a_1^2}{2} + \frac{a_2^2}{2} + \cdots \ldots + \frac{b_1^2}{2} + \frac{b_2^2}{2} + \cdots}}$$
$$= \sqrt{C_0^2 + \frac{1}{2}(C_1^2 + C_2^2 + \cdots \ldots)}$$

Where $C_0 = a_0$

And $C_n = \sqrt{a_n^2 + b_n^2}$ and it is called amplitude of harmonics.

POWER DUE TO NON-SINUSOIDAL VOLTAGES AND CURRENTS

The average power can be represented by

$$P_{avg} = \frac{1}{T}\int_0^T vi\, dt$$

The average power (non-sinusoidal) is the algebraic sum of the powers represented by corresponding harmonics of voltages and currents.

Consider

$$v = V_0 + \sum V_n \cos(n\omega t + \phi_n)$$
$$i = I_0 + \sum I_n \cos(n\omega t + \Psi_n)$$

and

$$V_{rms} = \sqrt{\mathbf{V_0^2} + \frac{\mathbf{V_1^2}}{\mathbf{2}} + \frac{\mathbf{V_2^2}}{\mathbf{2}} + \cdots \ldots}$$

$$I_{rms} = \sqrt{\mathbf{I_0^2} + \frac{\mathbf{I_1^2}}{\mathbf{2}} + \frac{\mathbf{I_2^2}}{\mathbf{2}} + \cdots \ldots}$$

∴ The average power can be represented by

$$P_{avg} = \frac{1}{T}\int_0^T vi\, dt$$

$$P_{avg} = \frac{1}{T}\int_0^T \left(V_0 + \sum V_n \cos(n\omega t + \phi_n)\right)\left(I_0 + \sum I_n \cos(n\omega t + \Psi_n)\right) dt$$

This product of two infinite series will yield

i) The product of constants.

ii) The product of a constant and cosine functions.

iii) The product of two cosine functions of different frequencies

iv) Cosine function squared.

After integration, the product of two constants is $V_0 I_0$ and the function squared with the limits applied appear as

$$\frac{V_n I_n}{2} \cos\left(\phi_n - \Psi_n\right)$$

While all other products upon the integration over the period T are zero.

Then the average power in standard form is given by

$$\mathbf{P_{avg}} = \mathbf{V_0 I_0} + \frac{\mathbf{V_1 I_1 \cos\theta_1}}{\mathbf{2}} + \frac{\mathbf{V_2 I_2 \cos\theta_2}}{\mathbf{2}} + \cdots \ldots$$

Where $\theta_1 = \phi_1 - \Psi_1$

$\theta_2 = \phi_2 - \Psi_2$

The above equation consists of DC, single frequency AC and also periodic non-sinusoidal waves.

POWER FACTOR

The power factor for non-sinusoidal wave is defined as the ratio of average power to the volt-ampere.

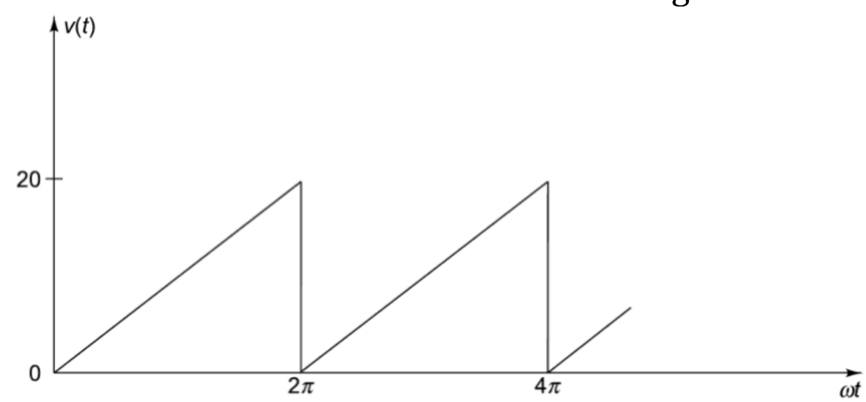
i.e. $\text{power factor} = \dfrac{\text{Average power}}{\text{Apparent power}}$

$$p.f. = \frac{V_0 I_0 + \frac{1}{2} V_1 I_1 \cos\theta_1 + \frac{1}{2} V_2 I_2 \cos\theta_2 + \cdots \ldots}{V_{rms}\, I_{rms}}$$

$$p.f. = \frac{V_0 I_0 + \frac{1}{2} V_1 I_1 \cos\theta_1 + \frac{1}{2} V_2 I_2 \cos\theta_2 + \cdots \ldots}{\sqrt{V_0^2 + \frac{V_1^2}{2} + \frac{V_2^2}{2} + \cdots \ldots}\ \sqrt{I_0^2 + \frac{I_1^2}{2} + \frac{I_2^2}{2} + \cdots \ldots}}$$

PROBLEMS

1) Find the Fourier series f the waveform shown in the fig.



SOL:

The waveform equation is given by

$$v(t) = \frac{20}{2\pi}\omega t$$

The average value of the waveform is

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} \frac{20}{2\pi}\omega t\, d(\omega t) = \left[20 \times \frac{(\omega t)^2}{2}\right]_0^{2\pi} = 10$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi}\left(\frac{20}{2\pi}\right)\omega t \cos n\omega t\; d(\omega t)$$

$$= \frac{20}{2\pi^2}\left[\frac{\omega t}{n}\sin n\omega t + \frac{1}{n^2}\cos n\omega t\right]_0^{2\pi}$$

$$= \frac{20}{2\pi^2 n^2}[\cos n2\pi - \cos 0]$$

$= 0$ for all integer values of n

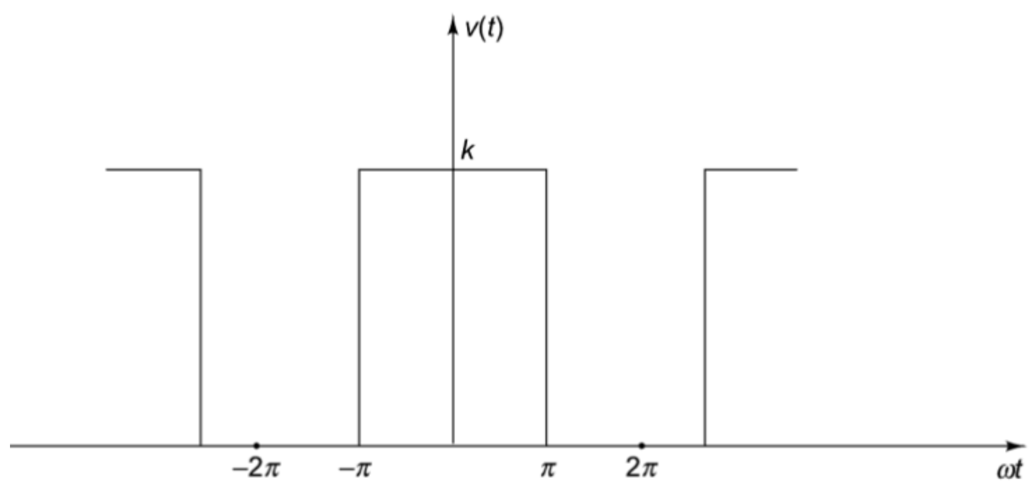
$$b_n = \frac{1}{\pi}\int_0^{2\pi}\left(\frac{20}{2\pi}\right)\omega t\, \sin n\omega t\, d(\omega t)$$

$$= \frac{20}{2\pi^2}\left[\frac{-\omega t}{n}\cos n\omega t + \frac{1}{n^2}\sin n\omega t\right]_0^{2\pi} = \frac{-20}{\pi n}$$

using these sine term coefficients and the average term, the series is

$$v(t) = 10 - \frac{20}{\pi}\sin \omega t - \frac{20}{2\pi}\sin 2\omega t - \frac{20}{3\pi}\sin 3\omega t$$

2) Find the fourier series of the function shown in the fig



SOL:

$$v(t) = \begin{cases} 0 & if \quad -2\pi < \omega t < -\pi \\ k & if \quad -\pi < \omega t < \pi \\ 0 & if \quad \pi < \omega t < 2\pi \end{cases}$$

the Fourier coefficients are

$$a_0 = \frac{1}{4\pi} \int_{-2\pi}^{2\pi} v(t)\, d(\omega t) = \frac{1}{4\pi} \int_{-\pi}^{\pi} k\, d(\omega t) = \frac{k}{2}$$

$$a_n = \frac{1}{2\pi} \int_{-2\pi}^{2\pi} v(t) \cos \frac{n\pi\omega t}{2} d(\omega t) = \frac{1}{2} \int_{-\pi}^{\pi} k \cos \frac{n\pi\omega t}{2} d(\omega t)$$

$$= \frac{2k}{n\pi} \sin \frac{n\pi}{2}$$

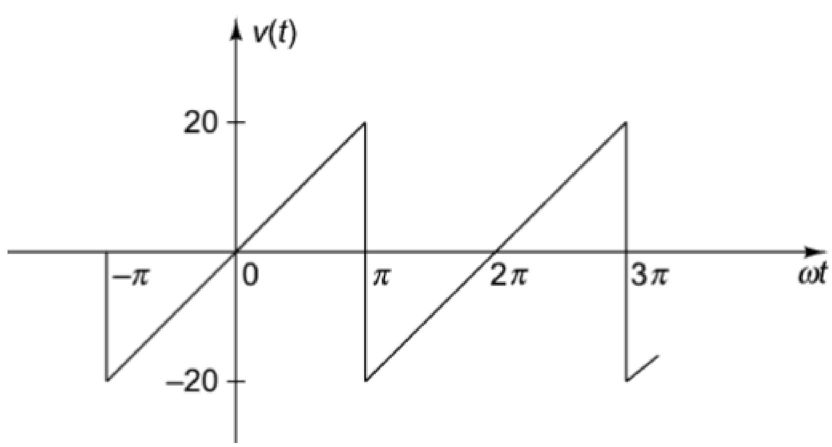
Thus, $a_n = 0$ if n is even and

$$a_n = \frac{2k}{n\pi} \quad \text{if} \quad n = 1, 5, 9, \ldots, a_n = \frac{-2k}{n\pi} \quad \text{if} \quad n = 3, 7, 11 \ldots$$

we also find that $b_n = 0$ for $n = 1,2...$ Hence, the result is

$$v(t) = \frac{k}{2} + \frac{2k}{\pi} \left[\cos \frac{\pi}{2} \omega t - \frac{1}{3} \cos \frac{3\pi}{2} \omega t + \frac{1}{5} \cos \frac{5\pi}{2} \omega t + \ldots \right]$$

3) Find the trignometric fourier series for the waveform shown in the fig.



SOL:

By inspection, the average value of the waveform $a_n = 0$. The waveform is odd and contains only sine terms. The expression of the waveform

$$v(t) = \left(\frac{20}{\pi}\right)\omega t \quad \text{for} -\pi < \omega t < \pi$$

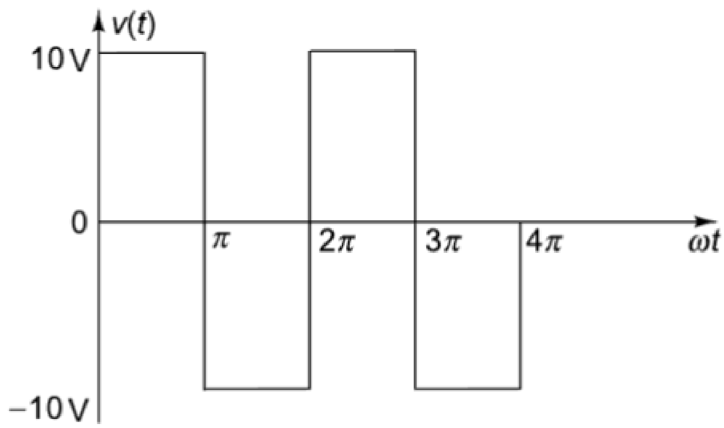
$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}\left(\frac{20}{\pi}\right)\omega t \sin n\omega t \; d(\omega t)$$

$$= \frac{20}{\pi^2}\left[\frac{1}{n^2}\sin n\omega t - \frac{\omega t}{n}\cos n\omega t\right]_{-\pi}^{\pi} = \frac{-40}{n\pi}\cos n\pi$$

The Fourier series is

$$v(t) = \frac{40}{\pi}\left\{\sin \omega t - \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t - \frac{1}{4}\sin 4\omega t + \dots\right\}$$

4) Find the complex Fourier sereis for the square waveform shown in the fig.



SOL:

The expression of the waveform

$$v(t) = -10 \text{ for } -\pi < \omega t < 0$$
$$= 10 \text{ for } 0 < \omega t < \pi$$

The average value of the wave is zero

$$c_n = \frac{1}{2\pi}\left\{\int_{-\pi}^{0}(-10)e^{-jn\omega t}\,d(\omega t)+\int_{0}^{\pi}(10)e^{-jn\omega t}\,d(\omega t)\right\}$$

$$= \frac{10}{2\pi}\left\{-\left[\frac{1}{-jn}e^{-jn\omega t}\right]_{-\pi}^{0}+\left[\frac{1}{-jn}e^{-jn\omega t}\right]_{0}^{\pi}\right\}$$

$$= \frac{10}{-j2\pi n}\left(-e^{0}+e^{jn\pi}+e^{-jn\pi}-e^{0}\right)$$

$$= \frac{j10}{n\pi}\left(e^{jn\pi}-1\right)$$

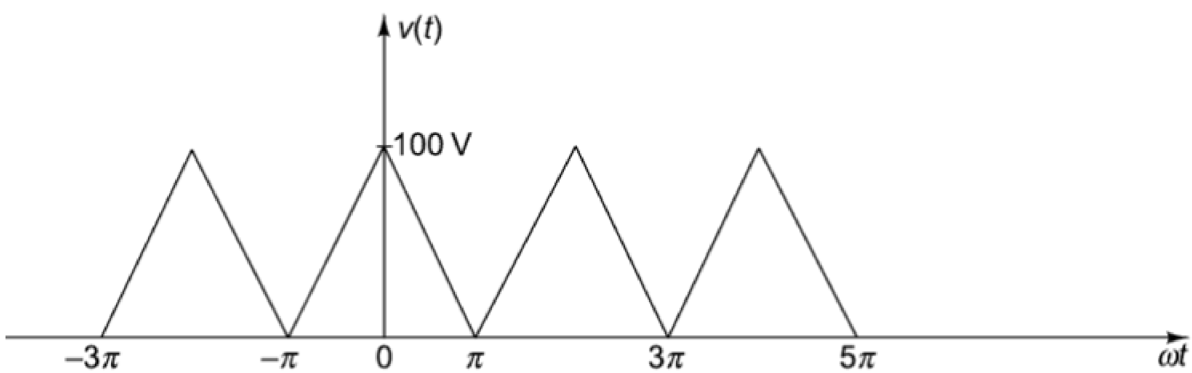
For n even $e^{jn\pi} = 1$ and $c_n = 0$

For n odd $e^{jn\pi} = -1$ and $c_n = \frac{-j(20)}{n\pi}$

The Fourier series is

$$x(t) = \cdots + j\frac{20}{3\pi}e^{-j3\omega t} - j\frac{20}{\pi}e^{-j\omega t} - j\frac{20}{\pi}e^{j\omega t} - j\frac{20}{3\pi}e^{j3\omega t}\cdots$$

5) Find the exponential fourier series for the waveform shown in the fig. and sketch the spectrum.



SOL:

The equation for the waveform shown in Fig. is given by

$$v(t) = 100 + \left(\frac{100}{\pi}\right)\omega t \quad \text{for } 0 < \omega t < \pi$$

$$v(t) = 100 - \left(\frac{100}{\pi}\right)\omega t \quad \text{for } -\pi < \omega t < 0$$

The waveform is even and the coefficients of A_n are pure real. By inspection, the average value is

$$C_0 = \frac{100}{2} = 50\,\text{V}$$

$$C_n = \frac{1}{2\pi}\left\{\int_{-\pi}^{0}\left[\left(100 + \frac{100}{\pi}\right)\omega t\right]e^{-jn\omega t}d(\omega t) + \int_{0}^{\pi}\left[\left(100 - \frac{100}{\pi}\right)\omega t\right]e^{-jn\omega t}d(\omega t)\right\}$$

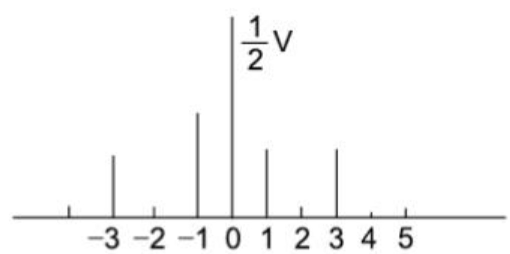
$$= \frac{100}{2\pi^2}\left\{\int_{-\pi}^{0}\omega t\, e^{-jn\omega t}d(\omega t) + \int_{0}^{\pi}(-\omega t)e^{-jn\omega t}d(\omega t) + \int_{-\pi}^{\pi}\pi e^{-jn\omega t}d(\omega t)\right\}$$

$$= \frac{100}{2\pi^2}\left\{\left[\frac{e^{-jn\omega t}}{(-jn)^2}(-jn\omega t - 1)\right]_{-\pi}^{0} - \left[\frac{e^{-jn\omega t}}{(-jn)^2}(-jn\omega t - 1)\right]_{0}^{\pi}\right\}$$

$$= \frac{100}{\pi^2 n^2}[1 - e^{jn\pi}]$$

$$C_n = 0 \qquad \text{for } n \text{ even}$$

$$C_n = \frac{200}{\pi^2 n^2} \qquad \text{for } n \text{ odd}$$

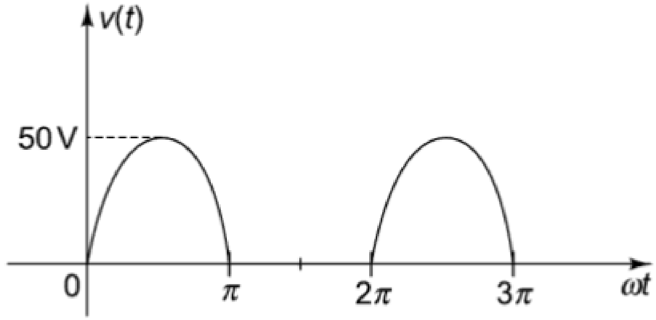


The exponential Fourier series is

$$v(t) = \cdots + \frac{200}{(-3\pi)^2}e^{-j3\omega t} + \frac{200}{(-\pi)^2}e^{-j\omega t} + \frac{100}{2} + \frac{200}{(\pi)^2}e^{j\omega t} + \frac{200}{(3\pi)^2}e^{j\omega t} + \ldots$$

The spectrum is shown in Fig.

6) Find the Trigonometric Fourier series for the half-wave rectified sinewave shown in the fig. and sketch the spectrum.



SOL:

The average value of the waveform

$$a_0 = \frac{1}{2\pi}\int_0^{\pi} 50\sin\omega t \, d(\omega t) = \frac{50}{2\pi}[-\cos\omega t]_o^{\pi} = \frac{50}{\pi}$$

The series contains both sine terms and cosine terms

$$a_n = \frac{1}{\pi}\int_0^{\pi} 50\sin\omega t \cos n\omega t \, d(\omega t)$$

$$= \frac{50}{\pi}\left[\frac{-n\sin\omega t \sin n\omega t - \cos n\omega t \cos\omega t}{-n^2+1}\right]_0^{\pi}$$

$$= \frac{50}{\pi(1-n^2)}(\cos n\pi + 1)$$

$$a_n = \frac{100}{\pi(1-n^2)} \quad \text{for } n \text{ even}$$

$$a_n = 0 \quad \text{for } n \text{ odd}$$

However, this expression is indeterminate for $n = 1$ and therefore we must integrate seperately for a_1,

$$a_1 = \frac{1}{\pi}\int_0^{\pi} 50\sin\omega t\cos\omega t\, d(\omega t) = \frac{50}{\pi}\int_0^{\pi}\frac{1}{2}\sin 2\omega t\, d(\omega t) = 0$$

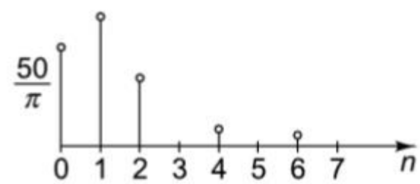
$$b_n = \frac{1}{\pi}\int_0^{\pi} 50\sin\omega t\sin n\omega t\, d(\omega t) = \frac{50}{\pi}\left[\frac{n\sin\omega t\cos n\omega t - \sin n\omega t\cos\omega t}{-n^2+1}\right]_0^{\pi} = 0$$

Here again the expression is indeterminate for $n = 1$, and b_1 is evaluated separately.

$$b_1 = \frac{1}{\pi}\int_0^{\pi} 50\sin^2\omega t\, d(\omega t) = \frac{50}{\pi}\left[\frac{\omega t}{2} - \frac{\sin 2\omega t}{4}\right]_0^{\pi} = 25$$

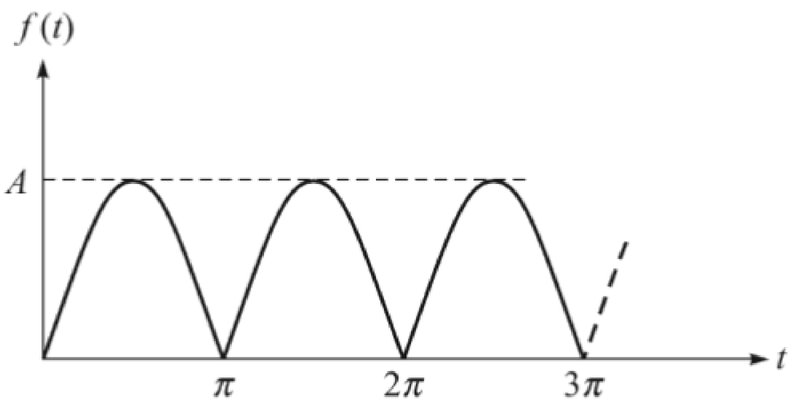
Then the Fourier series is

$$v(t) = \frac{50}{\pi}\left\{\begin{matrix} 1 + \frac{\pi}{2}\sin\omega t - \frac{2}{3}\cos 2\omega t \\ -\frac{2}{15}\cos 4\omega t - \frac{2}{35}\cos 6\omega t - \ldots \end{matrix}\right\}$$



The spectrum is shown in Fig.

7) Obtain the Fourier series expansion of the rectified sinewave shown in the fig.



SOL:

The equation of the half sine wave is $f(t) = A \sin\left(\frac{\omega t}{2}\right)$

Here $f(t) = f(-t)$, therefore, $f(t)$ is even and $b_n = 0$.

$$a_n = \frac{4}{T}\int_0^{T/2} f(t) \cos n\omega t \; dt = \frac{4A}{T}\int_0^{T/2} \sin\left(\frac{\omega t}{2}\right) \cos n\omega t \; dt$$

$$= \frac{2A}{T}\int_0^{T/2} 2 \sin\left(\frac{\omega t}{2}\right) \cos n\omega t \; dt$$

$$= \frac{2A}{T}\int_0^{T/2} \left[\sin\left(n+\frac{1}{2}\right)\omega t - \sin\left(n-\frac{1}{2}\right)\omega t\right]$$

$$= \frac{2A}{\omega T}\left[-\frac{\cos\left(n+\frac{1}{2}\right)\omega t}{\left(n+\frac{1}{2}\right)} + \frac{\cos\left(n-\frac{1}{2}\right)\omega t}{\left(n-\frac{1}{2}\right)}\right]_0^{T/2}$$

$$= \frac{A}{\pi}\left[-\frac{\cos\frac{1}{2}(2n+1)\pi - 1}{\frac{1}{2}(2n+1)} + \frac{\cos\frac{1}{2}(2n-1)\pi - 1}{\frac{1}{2}(2n-1)}\right]$$

$$= \frac{2A}{\pi}\left[\frac{1}{2n+1} - \frac{1}{2n-1}\right] = \frac{4A}{\pi(1-4n^2)}$$

$$\therefore \qquad a_0 = \frac{4A}{\pi}$$

$$\therefore \qquad f(t) = \frac{4A}{\pi} + \frac{4A}{\pi}\sum_{n=1}^{\infty} \frac{\cos n\omega t}{1-4n^2}$$

8) A complex wave is given by i(t) = 10+100 sinωt + 40 sin5ωt. Determine the average value and rms value of the complex wave.

SOL:

$$i(t) = 10 + 100\sin\omega t + 40\sin 5\omega t$$

$$I_{\text{avg}} = 10 \text{ A}$$

$$I_{\text{rms}} = \sqrt{(10)^2 + \left(\frac{100}{\sqrt{2}}\right)^2 + \left(\frac{40}{\sqrt{2}}\right)^2} = 76.81 \text{ A}$$

9) A series RL circuit with R = 10 **Ω** and L = 5H contains a current i(t) = 10 sin1000t + 5 sin 3000t + 3 sin 5000t. Find the effective value of voltage and the average power.

SOL:

$$i(t) = 10\sin 1000t + 5\sin 3000t + 3\sin 5000t$$

$$\omega = 1000 \text{ rad/s}$$

For fundamental harmonic,

$$R_1 = 10\ \Omega$$

$$X_{L_1} = \omega L = 1000 \times 5 = 5000\ \Omega$$

$$\mathbf{Z}_1 = 10 + j5000 = 5000\angle 89.89°\ \Omega$$

For third harmonic,

$$R_3 = 10\ \Omega$$

$$X_{L_3} = 3\omega L = 3 \times 1000 \times 5 = 15000\ \Omega$$

$$\mathbf{Z}_3 = 10 + j15000 = 15000\angle 89.96°\ \Omega$$

For fifth harmonic,

$$R_5 = 10\ \Omega$$

$$X_{L_5} = 5\omega L = 5 \times 1000 \times 5 = 25000\ \Omega$$

$$\mathbf{Z}_5 = 10 + j25000 = 25000\angle 89.98°\ \Omega$$

$$v(t) = (10)(5000)\sin(1000t + 89.89°) + (5)(15000)\sin(3000t + 89.96°)$$

$$+ 3(25000)\sin(5000t + 89.98°)$$

$$= 50000\sin(1000t + 89.89°) + 75000\sin(3000t + 89.96°)$$

$$+ 75000\sin(5000t + 89.98°)$$

$$V_{\text{eff}} = \sqrt{\left(\frac{V_{1m}}{\sqrt{2}}\right)^2 + \left(\frac{V_{2m}}{\sqrt{2}}\right)^2 + \left(\frac{V_{3m}}{\sqrt{2}}\right)^2} = \sqrt{\left(\frac{50000}{\sqrt{2}}\right)^2 + \left(\frac{75000}{\sqrt{2}}\right)^2 + \left(\frac{75000}{\sqrt{2}}\right)^2} = 82.92 \text{ kV}$$

$$P_{\text{avg}} = \frac{V_{1m}I_{1m}}{2}\cos\phi_1 + \frac{V_{2m}I_{2m}}{2}\cos\phi_2 + \frac{V_{3m}I_{3m}}{2}\cos\phi_3$$

$$= \frac{50000\times 10}{2}\cos(89.89°) + \frac{75000\times 5}{2}\cos(89.96°) + \frac{75000\times 3}{2}\cos(89.98°)$$

$$= 650.13 \text{ W}$$

10) The voltage given by v(t) = 100 cos 314t + 50 sin(1570t – 30^0) is applied to a circuit consisting of a resistor of 10 **Ω**, an inductor of 0.02 H and a capacitor of 50 μF. Determine the instantaneous current through the circuit. Also find the rms value of the voltage and current.

SOL:

$$v(t) = 100\cos 314t + 50\sin(1570t - 30°)$$

$$\omega = 314 \text{ rad/s}$$

For fundamental harmonic,

$$R_1 = 10\ \Omega$$

$$X_{L_1} = \omega L = 314\times 0.02 = 6.28\ \Omega$$

$$X_{C_1} = \frac{1}{\omega C} = \frac{1}{314\times 50\times 10^{-6}} = 63.69\ \Omega$$

$$\mathbf{Z}_1 = 10 + j6.28 - j63.69 = 58.27\angle -80.12°\ \Omega$$

For fifth harmonic,

$$R_5 = 10\ \Omega$$

$$X_{L_5} = 5\omega L = 5\times 314\times 0.02 = 31.4\ \Omega$$

$$X_{C_5} = \frac{1}{5\omega C} = \frac{1}{5\times 314\times 50\times 10^{-6}} = 12.74\ \Omega$$

$$\mathbf{Z}_2 = 10 + j31.4 - j12.74 = 21.17\angle 61.81°\ \Omega$$

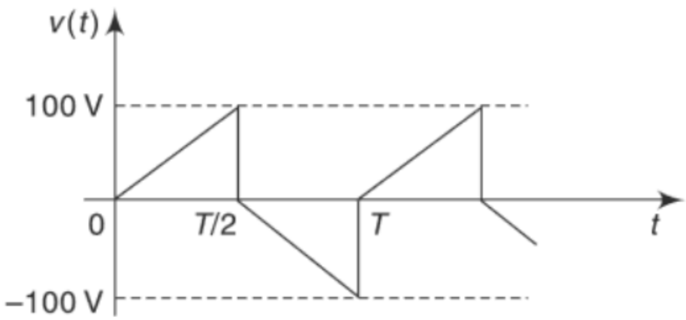
$$i(t) = \frac{100}{58.27}\cos(314t + 80.12°) + \frac{50}{21.17}\sin(1570t - 30° - 61.81°)$$

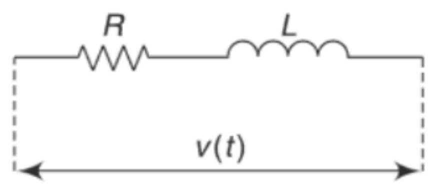
$$= 1.72\cos(314t + 80.12°) + 2.36\sin(1570t - 91.81°)$$

$$V_{\text{rms}} = \sqrt{\left(\frac{V_{1m}}{\sqrt{2}}\right)^2 + \left(\frac{V_{2m}}{\sqrt{2}}\right)^2} = \sqrt{\left(\frac{100}{\sqrt{2}}\right)^2 + \left(\frac{50}{\sqrt{2}}\right)^2} = 79.06 \text{ V}$$

$$I_{\text{rms}} = \sqrt{\left(\frac{I_{1m}}{\sqrt{2}}\right)^2 + \left(\frac{I_{2m}}{\sqrt{2}}\right)^2} = \sqrt{\left(\frac{1.72}{\sqrt{2}}\right)^2 + \left(\frac{2.36}{\sqrt{2}}\right)^2} = 2.06 \text{ A}$$

11) Determine the Fourier series of repetitive waveform of figure as shown in the fig. upto 5th harmonic, when the time of repetition, T = 20 ms. Calculate fundamental frequency current in the circuit of figure as shown when R = 10 **Ω** and L = 0.0318 H with voltage transform of the waveform.





SOL:

The wave is having half wave symmetry.

$a_n = b_n = 0$; for n even ; and

For n odd,

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\sin n\omega t\, dt$$

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin n\omega t\, dt$$

and $\quad a_0 = 0$

Now, $$v(t) = \frac{200}{T}t;\ 0 \le t \le \frac{T}{2}$$

$\therefore$ $$a_n = \frac{4}{T}\int_0^{T/t} \frac{200}{T} t \cos n\omega t \, dt$$

$$= \frac{800}{T^2}\left[\frac{t \sin nwt}{n\omega} - \int \frac{\sin n\omega t}{n\omega} dt\right]$$

$$= \frac{800}{T^2}\left[\frac{T}{2} \times \frac{\sin n\pi}{n\omega} + \frac{\cos n\omega t}{n^2\omega^2}\Big|_0^{T/2}\right]$$

$$= \frac{800}{n^2\omega^2 T^2}\left[\cos n\pi - 1\right]$$

$$= \frac{800}{n^2 4\pi^2}(-2)$$

$$= -\frac{400}{n^2\pi^2}$$

$$b_n = \frac{4}{T}\int_0^{\frac{T}{2}} \frac{200}{T} t \sin n\omega t dt = \frac{200}{n\pi}$$

$$\therefore \quad v(t) = -\frac{400}{\pi^2}\left(\cos\omega t + \frac{1}{3^2}\cos 3\omega t + \frac{1}{5^2}\cos 5\omega t + ...\right) + \frac{200}{\pi}\left(\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + ...\right)$$

The fundamental frequency voltage is

$$V_f = \left(\frac{200}{\pi}\sin\omega t - \frac{400}{\pi^2}\cos\omega t\right) = \frac{400}{\pi^2}\sqrt{\left(\frac{\pi}{2}\right)^2 + 1}$$

Impedance, $Z = (R + j\omega L) = 10 + j\omega(0 \cdot 0318)$

Current due to fundamental frequency,

$$I_f = \frac{V_f}{Z} = \frac{400}{\pi^2(10 + j0\cdot0318\omega)}\left(\frac{\pi}{2}\sin\omega t - \cos\omega t\right)$$

or $$I_f = \frac{400}{\pi^2}\sqrt{\left(\frac{\pi}{2}\right)^2 + 1} \times \frac{1}{\sqrt{(10)^2 + (0\cdot0318\omega)^2}} \angle \tan^{-1}\frac{0.0318\omega}{10}$$

Here, $$\omega = \frac{2\pi}{T} = \frac{2\pi}{20\times10^{-3}} = 100\pi \text{ rad/s}$$

Patting this value,

$$\therefore \quad I_f = 5\cdot33\angle -44.9^\circ$$

$$\therefore \quad I_{f(\text{rms})} = \frac{5\cdot33}{\sqrt{2}}\text{ A} = 3\cdot76 \text{ A}$$

12) A complex power e(t)=100 sin ωt + 30 sin3ωt + 20 sin5ωt
Where ω=100rad/sec. If this voltage is applied to a load of 10Ω in series with 0.01H. Find the current, average power and power factor of the circuit.

SOL :

Given ω=100rad/sec, R=10Ω and L= 0.01H

Input voltage, e(t)=100 sin 100t + 30 sin300t + 20 sin500t

The input voltage contains fundamental, third and fifth harmonic components. Hence the current equation will be

$$i(t) = \frac{100}{Z_1}\sin 100t + \frac{30}{Z_2}\sin 300t + \frac{20}{Z_3}\sin 500t \qquad \text{----------(1)}$$

Where

$Z_1 = R + jX_L$

$= R + j\omega L$

$= 10 + j[(100 \times 0.01)]$

$= 10 + j\,1$

$= 10.05\ e^{+j\tan^{-1}\left(\frac{1}{10}\right)}$

$$\mathbf{Z_1 = 10.05\ e^{+j5.7^0}\ \Omega}$$

$Z_2 = R + jX_L$

$= R + j\omega L$

$= 10 + j[(300 \times 0.01)]$

$= 10 + j\,3$

$$= \sqrt{10^2 + 3^2}\ e^{+j\tan^{-1}\left(\frac{3}{10}\right)}$$

$$\mathbf{Z_2 = 10.44\ e^{+j16.7^0}\ \Omega}$$

$Z_3 = R + jX_L$

$= R + j\omega L$

$= 10 + j[(500 \times 0.01)]$

$= 10 + j\,5$

$$= \sqrt{10^2 + 5^2}\ e^{+j\tan^{-1}\left(\frac{5}{10}\right)}$$

$$\mathbf{Z_3 = 11.18\ e^{+j26.6^0}\ \Omega}$$

From eq(1),

$\therefore$ current $i(t) = \frac{100}{10.05}\, e^{-j5.7^0} \sin 100t + \frac{30}{10.44}\, e^{-j16.7^0} \sin 300t + \frac{20}{11.18}\, e^{-j26.6^0} \sin 500t$

$$\mathbf{i(t) = 9.95\ e^{-j5.7^0} \sin 100t + 2.9\ e^{-j16.7^0} \sin 300t + 1.79\ e^{-j26.6^0} \sin 500t}$$

$\therefore$ Effective value of current, $I_{eff} = I_{rms} = \sqrt{\frac{I_1^2}{2} + \frac{I_3^2}{2} + \frac{I_5^2}{2}}$

$$= \sqrt{\frac{9.95^2}{2} + \frac{2.9^2}{2} + \frac{1.79^2}{2}}$$

$$\mathbf{I_{eff} = 7.44A}$$

$\therefore$ Effective value of voltage, $V_{eff} = V_{rms} = \sqrt{\frac{V_1^2}{2} + \frac{V_3^2}{2} + \frac{V_5^2}{2}}$

$$= \sqrt{\frac{100^2}{2} + \frac{30^2}{2} + \frac{20^2}{2}}$$

$$\mathbf{V_{eff} = 75.17V}$$

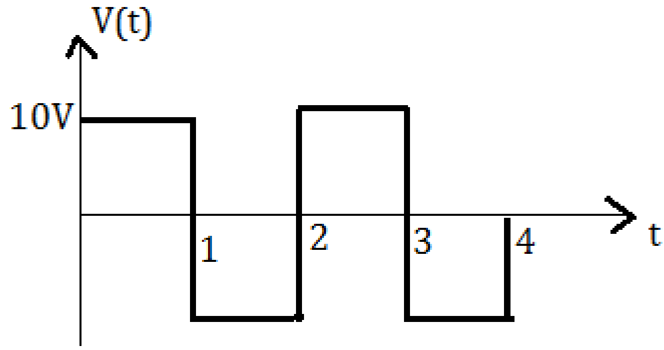
Average power, $P_{avg} = \frac{V_1 I_1 \cos\theta_1}{2} + \frac{V_3 I_3 \cos\theta_3}{2} + \frac{V_5 I_5 \cos\theta_5}{2}$

$$= \frac{1}{2} \times 100 \times 9.95 \times \cos 5.7 + \frac{1}{2} \times 30 \times 2.9 \times \cos 16.7 + \frac{1}{2} \times 20 \times 1.79 \times \cos 26.6$$

$$\mathbf{P_{avg} = 552.7\ Watts}$$

Power factor $= \frac{\text{Average power}}{V_{eff}\, I_{eff}} = \frac{552.7}{(7.44)(75.17)} = 0.988$

13) Find the trigonometric form of the following voltage waveform and hence compute power and power factor of the load if voltage is applied to a series RL circuit with R=1Ω and L=1H.



SOL:

The given waveform is odd function.

$\therefore a_n = a_0 = 0$

$$b_n = \frac{4}{T}\int_0^{\frac{T}{2}} 10 \sin n\omega t \, dt$$

From the waveform, T=2 sec

$$b_n = 2\int_0^{1} 10 \sin n\omega t \, dt$$

$$= 20\left[-\frac{\cos n\omega t}{n\omega}\right]_0^1$$

$$b_n = -\frac{20}{n\omega}(\cos n\omega - 1)$$

We have, $\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$

$$\therefore b_n = -\frac{20}{n\pi}(\cos n\pi - 1)$$

$$b_n = -\frac{20}{n\pi}(-1-1), \text{ n is odd}$$

$$\mathbf{b_n = \frac{40}{n\pi}}, \text{ n is odd i.e. } n=1,3,5,7,\ldots\ldots\ldots$$

$$\therefore V(t) = \sum_{n=1}^{\infty} b_n \sin \omega t$$

$$V(t) = \sum_{n=odd} \frac{40}{n\pi} \sin\omega t = \sum_{n=odd} \frac{40}{n\pi} \sin n\pi t$$

$$\therefore V(t) = \frac{40}{\pi}\left[\sin\pi t - \frac{1}{3}\sin 3\pi t + \frac{1}{5}\sin 5\pi t - \cdots\cdots\cdots\right]$$

The inductor does not allow higher order harmonics.

$$\therefore V(t) = \frac{40}{\pi}\left[\sin\pi t - \frac{1}{3}\sin 3\pi t + \frac{1}{5}\sin 5\pi t\right]$$

$$\therefore \text{Current, } i(t) = \frac{40}{\pi}\left[\frac{1}{Z_1}\sin\pi t - \frac{1}{3Z_2}\sin 3\pi t + \frac{1}{5Z_3}\sin 5\pi t\right]$$

$$Z_1 = R + jX_L = R + j\omega L = 1+j\pi = \sqrt{1^2+\pi^2}\,\angle\tan^{-1}\pi = 3.3\angle 72.3\ \Omega$$

$$Z_2 = R + j\omega L = 1+j3\pi = \sqrt{1^2+(3\pi)^2}\,\angle\tan^{-1}3\pi = 9.5\angle 83.9\ \Omega$$

$$Z_3 = R + j\omega L = 1+j5\pi = \sqrt{1^2+(5\pi)^2}\,\angle\tan^{-1}5\pi = 15.74\angle 86.36\ \Omega$$

$\therefore$ Current,

$$i(t) = \frac{40}{\pi}\left[\frac{1}{3.3}\angle-72.3\sin\pi t - \frac{1}{3\times 9.5}\angle-83.9\sin 3\pi t + \frac{1}{5\times 15.74}\angle-86.36\sin 5\pi t\right]$$

$$i(t) = [3.86\angle-72.3\sin\pi t - 0.447\angle-83.9\sin 3\pi t + 0.16\angle-86.36\sin 5\pi t]$$

$$\mathbf{I_{rms}} = \sqrt{\frac{I_1^2}{2}+\frac{I_3^2}{2}+\frac{I_5^2}{2}} = \sqrt{\frac{3.86^2}{2}+\frac{0.447^2}{2}+\frac{0.16^2}{2}} = \mathbf{2.75A}$$

$$V(t) = 12.7\sin\pi t - 4.24\sin 3\pi t + 2.55\sin 5\pi t$$

$$\mathbf{V_{rms}} = \sqrt{\frac{V_1^2}{2}+\frac{V_3^2}{2}+\frac{V_5^2}{2}} = \sqrt{\frac{12.7^2}{2}+\frac{4.24^2}{2}+\frac{2.55^2}{2}} = \mathbf{9.64}$$

$$\text{Average power, Pavg} = \frac{V_1 I_1\cos\theta_1}{2} + \frac{V_3 I_3\cos\theta_3}{2} + \frac{V_5 I_5\cos\theta_5}{2}$$

$$= \frac{1}{2}\times 12.7\times 3.86\times\cos 72.3 + \frac{1}{2}\times(-4.24)\times(-0.447)\times\cos 83.9 + \frac{1}{2}\times 2.55\times 0.16\times\cos 86.36$$

$$\mathbf{P_{avg} = 7.57\ Watts}$$

$$\textbf{Power factor} = \frac{\text{Average power}}{V_{rms}\,I_{rms}} = \frac{7.57}{(9.64)(2.75)} = \mathbf{0.29}$$